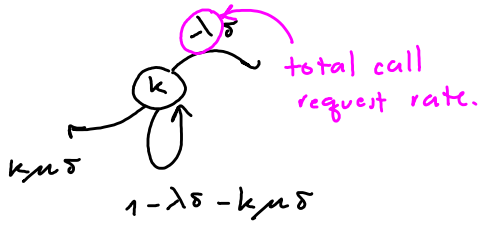
Friday, January 07, 2011
10:37 AM

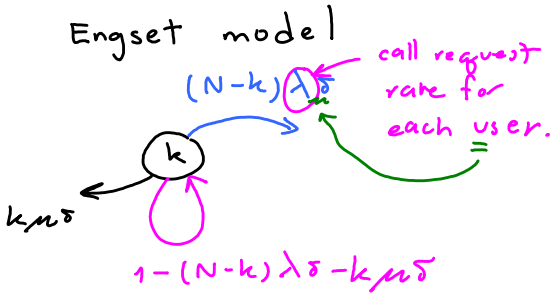
Lecture 14 (Jan 7)

Erlang B v.s. Engset Model

Erlang B model



Engset model



If you have m channels, then need to do this m+1 times.
(for state 0, 1, ..., m)

Steady-state probabilities

$$p_k = \frac{A^k/k!}{\sum_{i=0}^{m} A^i/i!}$$

time congestion $= p_m$

call congestion $= p_m$

call blocking probability

Steady-state probabilities

$$p_k = \frac{\binom{N}{k} A^k}{\sum_{i=0}^{m} \binom{N}{i} A^i}$$

Truncated Binomial

$0 \le k \le m$

time congestion $= p_m$

call congestion $(\neq p_m)$

$$= \frac{(N-m)p_m}{\sum_{k=0}^{m} (N-k)p_k}$$

Quiz 2

Erlang B: $\lambda = 10 \frac{\text{calls}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ min}} = \frac{1}{6} \text{ calls/min}$

$\frac{1}{\mu} = 12$ mins

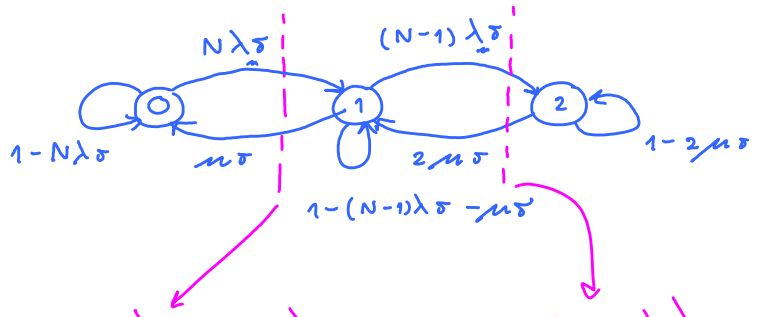
$A = \lambda \times \frac{1}{\mu} = \frac{1}{6} \times 12^2 = 2$ Erlang.

Engset: $\lambda_u = 2$ calls/hour

$\frac{1}{\mu} = 12$ mins.

$A_m = \lambda_m \times \frac{1}{\mu} = \frac{2}{5}$ Erlang

$N = 5$



$p_0(N\lambda\delta) = p_1\mu\delta$ $\qquad$ $p_1(N-1)\lambda\delta = p_2 2\mu\delta$

$p_1 = NA_m p_0$ $\qquad$ $p_2 = \frac{N-1}{2} A_m p_1$

$= \frac{N(N-1)}{2} A_m^2 p_0$

$p_0 + p_1 + p_2 = 1 \Rightarrow p_0 = \frac{1}{1 + NA_m + \frac{N(N-1)}{2} A_m^2}$

call blocking probability $= \frac{(N-2)p_2}{(N-0)p_0 + (N-1)p_1 + (N-2)p_2}$

Remarks

(1) If we keep total rate λ constant,
let $N \to \infty$

call blocking probability in Engset model $\longrightarrow$ Erlang B model

(2) If $N \leq m$

call blocking prob. in Engset model $= 0$.